

1 **DYNAMIC PASSENGER ASSIGNMENT CHALLENGES**

2

3

4

5 Lisa Zorn
6 Metropolitan Transportation Commission
7 Bay Area Metro Center
8 375 Beale Street, Suite 800
9 San Francisco, CA 94105-2066
10 415-778-6644
11 Main Fax Number: 415-536-9800
12 lzorn@mtc.ca.gov

13 [corresponding author]

14

15 Elizabeth Sall
16 UrbanLabs LLC
17 4405 4th Ave NE
18 Seattle WA 98105
19 919-302-0265
20 easall@gmail.com

21

22 Word count: 4,662 + 1 table + 1 figure = 5,162

1 ABSTRACT

2 Fast-Trips is a multi-agency effort to develop and implement a dynamic transit passenger assignment model for
3 travel demand forecasting. It represents each transit passenger individually, along with their demand attributes and a
4 target origin departure or destination arrival time. On the supply side, each transit vehicle trip and the capacity of
5 that vehicle is represented. In the course of this project, the Fast-Trips team has encountered a number of challenges
6 in developing a computationally feasible, mathematically consistent, unbiased model. This paper documents the
7 challenges encountered, the work that has been done in these areas so far, and the strategy the Fast-Trips team will
8 pursue in the near-term. The research areas consist of: the modeling of non-link additive variables such as fares and
9 qualitative variables; accounting for the overlap between paths in terms of the impact on path probability
10 calculations; modeling the traveler's approach to deciding to board a transit vehicle at a stop where multiple lines
11 serve a common route; and accounting for stochastic vehicle arrivals. The goal of this paper is to give an overview
12 of these challenges from an implementation and application (rather than purely theoretic) standpoint to encourage
13 research and collaboration in solving them.
14

1 **INTRODUCTION**

2 **What is this paper about?**

3 Over the past year and a half, a team from three public agencies plus several consultants have been engaged in the
 4 Fast-Trips Implementation project, with the goal of developing a dynamic transit passenger assignment tool to assist
 5 in travel demand modeling and forecasting. Fast-Trips takes as input a transit network and transit passenger demand
 6 and finds paths for each transit passenger trip, assigning those passenger trips to transit vehicles. However, in the
 7 course of the project thus far, the team has come across a number of problems that were not anticipated at the outset.
 8 This paper is not meant to describe the solutions to these problems, because as it turns out, they're difficult and the
 9 solutions are not clear. Rather, this paper is meant to gather them all together in a single place and document them in
 10 a way that is, hopefully, useful to aiding and encouraging research into solving these problems.

11 When the Fast-Trips Implementation project initially kicked off, the project schedule was comprised
 12 primarily of the development of data standards and input preparation, software development, and transit route choice
 13 estimation. As such, the problems described in this paper arose during research into model estimation and
 14 implementation.

15 **Background and Terminology**

16 To clarify terminology, henceforth in this paper, the
 17 universal set of all paths from an origin to a
 18 destination for a particular individual with a target
 19 arrival or destination time will be called U . **Path set**
 20 **generation** is then the process of generating a
 21 subset, M , of that universal set, where M may be the
 22 same as U . If path set generation is stochastic, then
 23 M is better described as M_n where n is a random
 24 variable.

25 The step following path set generation is
 26 typically **choice set formation** or **path set pruning**,
 27 which may remove "bad" paths from the path set.
 28 This results in a path set C_n (which may be equal to
 29 M or M_n). In probabilistic choice set models, each
 30 non-empty subset of the path set is evaluated as a
 31 choice set and the probability of that choice set is
 32 calculated. However, this adds considerable
 33 complexity for real applications, so it has not been
 34 used in practice.

35 The **route choice model** follows, assigning
 36 a probability to each path i in the choice set. Note
 37 that path set pruning may utilize the route choice
 38 model assumptions to determine what it means to be
 39 a bad path to prune. Finally, once a route choice set
 40 has been defined with probabilities associated for
 41 each path, the passenger demand can be loaded or
 42 **assigned** back onto the transit network.

43 In the traffic route choice context, the
 44 difference between a dynamic versus a static
 45 model has to do with the level of temporal
 46 aggregation applied to both supply and demand. In static traffic assignment, the volume on a link represents traffic
 47 volume over a relatively long time period (30 minutes or more) so a fundamental assumption of the model is that
 48 link inflow equals to link outflow over this unit of time. In contrast, in a dynamic model, the unit of time
 49 aggregation is smaller (typically 5 minutes or less), and this flow equality need not hold true. Thus, each link may be
 50 defined by its own fundamental diagram which describes how congestion propagates from the exit node to the
 51 entrance node, and link congestion, queuing and spill-back can be modeled (1).

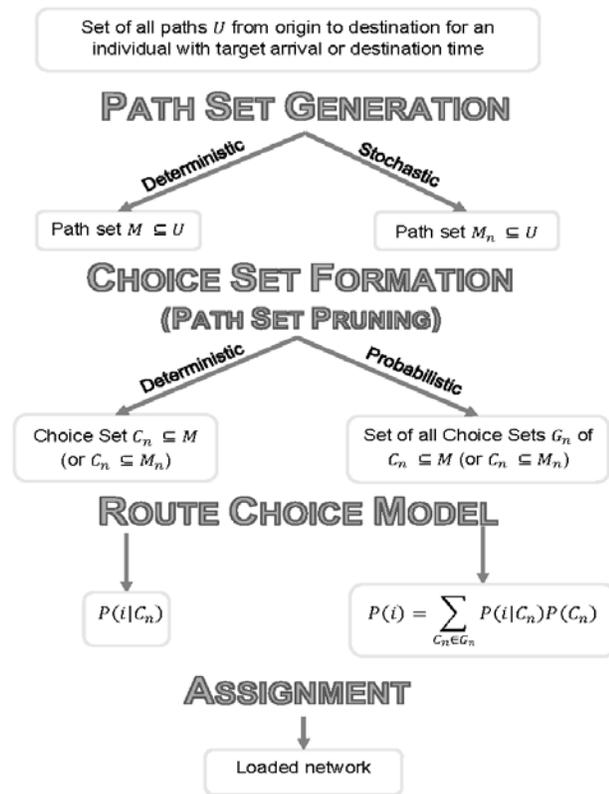


FIGURE I. Transit Assignment Model process.

1 In transit route choice and assignment, a **static** transit assignment model similarly represents multiple runs
2 of a single transit route with a single link by assuming a constant level of service across an aggregated, several-hour
3 time period. A **dynamic** transit assignment model, by contrast, represents each transit vehicle run individually and
4 on a specific schedule. As in the traffic context, this enables more accurate modeling of transit vehicle capacity
5 constraints, queuing at stops, and transfer situations (2).

6 **What is Fast-Trips?**

7 FAST-TrIPs (which stands for Flexible Assignment and Simulation Tool for Transit and Intermodal Passengers)
8 started out as an open source dynamic transit assignment model (including path set generation, path set pruning and
9 route choice) developed at the University of Arizona and the University of Texas at Austin by Khani, Hickman and
10 Noh (3). It utilized a trip-based hyperpath model (TBHP) to generate a set of paths with low generalized costs (4,5).
11 The TBHP algorithm is basically a variant of Dijkstra's Algorithm for finding a shortest path with a few variations.
12 At a high level, TBHP differs from Dijkstra's in that, rather than keeping track of a single path by tracking the
13 previous node, it keeps track of multiple paths by way of multiple options for previous nodes. It must therefore also
14 keep track of multiple costs, and combine them into a single aggregate cost, or the hyperlink cost. TBHP is a
15 stochastic path set generation algorithm because each hyperlink represents a number of actual links which are
16 chosen probabilistically when paths are enumerated. Note that TBHP can be formulated as a frequency-based or
17 schedule-based model, and FAST-TrIPs applied it to a schedule-based network (6).

18 In 2014, three agencies, Metropolitan Transportation Commission (MTC), San Francisco County
19 Transportation Authority (SFCTA), and Puget Sound Regional Council (PSRC), applied for and won a federal grant
20 through the SHRP-2 program to extend this work to develop a tool to analyze transportation investments. Fast-Trips
21 is a pivot of the original FAST-TrIPs code base, but with several new requirements which will be described below.
22 For the remainder of this paper, *Fast-Trips* (with only F and T capitalized) will refer to the SHRP-2 project, while
23 the *original FAST-TrIPs* (all caps except for r and the trailing s) will refer to the original project developed by
24 Khani, Hickman and Noh.

25 **MOTIVATION AND BACKGROUND**

26 **What Problems is Fast-Trips Meant to Solve?**

27 The original implementation, FAST-TrIPs, solved several problems that are important to the project stakeholders
28 that earlier transit route choice tools could not. These included the ability to represent a more nuanced schedule of
29 transit vehicle trips and the resulting ridership, including

- 30 ● network link travel times that could be informed by dynamic traffic assignment models;
- 31 ● passenger queueing at transit stops;
- 32 ● stop dwell times that could be a function of boards, alights and crowding;
- 33 ● the ability to consider a set of transit paths for each passenger, rather than a single best path, in order to
34 represent the benefit of having more than one option; and
- 35 ● transit vehicle capacities, which could result in bumped passengers, affecting transit path quality.

36 In the Fast-Trips Implementation Project for the SHRP-2 Implementation Assistance Program, stakeholders added a
37 few more requirements to support long-term planning. These included:

- 38 ● Heterogeneity of riders: different types of riders can value (or devalue) different aspects of a transit trip
39 differently. For example, older travelers may be more bothered by extreme crowding or longer walks. By
40 representing this heterogeneity of riders, Fast-Trips could better assess the demographics of travelers that
41 would benefit most from a planned project.
- 42 ● Passengers affect transit: boardings, alightings and crowding often affect transit vehicle dwell times. The
43 relationship between transit dwell times and other variables should be fully configurable based on transit
44 vehicle type.
- 45 ● Transit affects passenger experiences: some people will get seats while others will not be able to board
46 crowded vehicles and wait longer. Some people will miss transfers while others may ride a few stops in the
47 wrong direction to get a seat on a crowded line.
- 48 ● Transit reliability: Missed transfers and reliability issues affect how people perceive the quality of transit.

1 **What exists already?**

2 In practice, transit route choice and assignment models that are in active use today tend to be static models included
3 as part of commercial travel modeling software packages. INRO's Emme has the most documentation in terms of
4 the underlying research behind the model, and several modeling innovations included in this model are referenced in
5 sections below (7,8). PTV Group's VISUM model includes headway-based and time-table-based assignment, as
6 well as non-additive fares, but it's not clear how these are implemented (9). Caliper's website says that TRANSCAD
7 includes "the broadest set of transit assignment methods including some innovative methods not found in other
8 packages. These include a stochastic user equilibrium method that deals with multiple service alternatives, vehicle
9 capacity, and optionally with dwell time and user's value of time", although the implementation details behind these
10 innovations are not published (10). Citilab's Cube Voyager also comes with two static transit route
11 choice/assignment models, TRNBUILD and Public Transport, the internal details of which are not published either.
12 In order to test the effects of policies affecting transit capacity increases, the San Francisco County Transportation
13 Authority developed a transit assignment model on top of TRNBUILD; this represented capacity constraints and
14 passenger-based transit delays in a limited way by programmatically modifying the inputs to TRNBUILD to deter
15 passengers from crowded vehicles and adjust schedules (11).

16 Beyond practice, there has been much research into dynamic transit assignment modeling and much of that
17 research is discussed below in greater detail. In broad strokes, most of this is based on lowest-cost path search using
18 network labeling like Fast-Trips. However, the Fast-Trips team has also been looking at a novel approach called
19 Recursive Logit, which corresponds to a sequence of link choices that together form a path choice. This approach is
20 unique in that it avoids path enumeration, and so it can be consistently estimated and used for prediction without
21 limiting the path choice set (12). Although this framework is promising, the Fast-Trips team is still planning to
22 develop, enhance and test the hyperpath-based model from the original FAST-Trips, because it is closer to practice-
23 ready.

24 **WHAT'S HARD ABOUT TRANSIT ROUTE CHOICE**

25 This section discusses a variety of theoretical hurdles that the Fast-Trips Implementation team struggled with that
26 are all deemed necessary to address in order to successfully address stakeholder needs.

27 **Non-additive variables**

28 *Why is this important?*

29 Computing non-additive variables is desirable to transit route choice, primarily because of fare systems which can
30 encompass complicated transfer rules and discounts. For example, a passenger could take a bus to rail to bus trip,
31 and the second bus trip may be a free transfer. Representing this non-additive fare is important for understanding the
32 monetary cost of a transit trip, which is critical to being able to analyze the effects of fare policies on ridership
33 especially for populations that are cost-sensitive. Additionally, other path metrics, such reliability or qualitative
34 metrics, may not be perceived to be link-additive to travelers.

35 *What's the problem?*

36 Computing these non-additive variables is typically difficult in path-finding algorithms including TBHP because
37 they walk a path or a set of paths but have uncertainty in the links that precede the current link. In particular, in the
38 TBHP algorithm, since each hyperlink represents a set of possible transit links, the algorithm cannot determine with
39 certainty what all the previous links are when tracing the shortest paths. Even if it were to use an expected value of
40 fare, going back to links earlier in the chain becomes problematic to track, because doing so would require large
41 amounts of processing time and/or memory. This problem also exists in the recursive logit modeling framework.

42 *What work has been done already?*

43 Lo et al. (13) created a mode-transfer state diagram and then multiplied out the network to create a "State-
44 augmented multi-modal network" to enable transfer tracking. Constantin and Florian (8) propose keeping an
45 additional component for the traveler state during strategy-based transit path-finding. They call this component the
46 Journey Level, and it's combinatorial on the number of modes which need to be tracked. For example, if there are
47 free transfers within system A buses and within system B light rail, then there four journey levels: no payment, paid
48 system A bus fare, paid system B light rail, and paid both. Constantin and Florian find that so long as the fare levels
49 are limited, this results in an acceptable efficiency, and this strategy has been incorporated into INRO's Emme
50 transportation forecasting software. A similar method could be explored for Fast-Trips for non-additive fare

1 structures, although since Fast-Trips already has computation-time challenges due to being schedule-based, these
2 approaches may not be feasible.

3 **Accounting for route overlap**

4 *What's the problem?*

5 Route overlap is a well-known problem in route choice modeling, and it refers to the fact that travelers considering a
6 route from a choice set do not view the routes as independent when the routes have overlap. As a transit example,
7 suppose a traveler has three transit route options from point A to B:

- 8 • option 1: bus trip B1 from stop S1 to stop S2,
- 9 • option 2: bus trip B1 from stop S1 to stop S3, and
- 10 • option 3: light rail trip L.

11 Suppose that the traveler's destination is situated between stop S2 and S3 such that the overall generalized
12 cost is the same regardless of which stop they alight. Because of the overlap between options 1 and 2, the traveler
13 would not view them as being completely separate options, and would not value this choice set as having three full
14 paths. Thus, the modeling framework and route choice estimation needs to take this into account.

15 *Why is this important?*

16 If the route choice estimation does not take route choice overlap into account, it will bias estimation. Even with an
17 unbiased estimation, route choice must incorporate choice overlap or it will produce inaccurate path probabilities.
18 To illustrate, suppose that option 3 happens to have the same generalized cost as options 1 and 2. A modeling
19 framework that ignores route choice overlap would assign equal probabilities to each of the three options instead of
20 splitting the probabilities between the bus option and the light rail option. Transit route choice has additional
21 complications over a strictly roadway-network based route choice model in that option 1 and option 2 are likely still
22 perceived as overlapping if they are the same bus route but different vehicle trips running a few minutes apart.
23 However, this overlap may also be perceived as a benefit because high frequency service can compensate for
24 unreliability issues with any individual vehicle trip.

25 *What work has been done already?*

26 Cascetta et al. (14) formulated the C-Logit model which added a "commonality factor" penalty to the utility
27 structure of the multinomial logit model. Similarly, Ben-Akiva and Bierlaire (15) formulated a path size correction
28 to penalize the utility of overlapping paths, defining the Path Size Logit model. Frejinger et al. (16) noted that the
29 path size correction attribute should also correct for the sampling protocol that generated the path set, proposing a
30 new sampling approach along with an Expanded Path Size factor. While the C-Logit and Path Size Logit models
31 have been primarily applied to auto and truck route choice models, Hoogendoorn-Lanser (17) explored three
32 variations of overlap definitions in the context of transit trips using Path Size Logit, finding that defining overlap in
33 terms of number of legs performs better than defining overlap in overlapping travel time or travel distance.

34 **Common Lines Problem**

35 *What's the problem?*

36 The common lines problem was first formulated by Chriqui and Robillard (18) in the context of static transit
37 assignment. The problem is basically that a transit route choice model must account for the fact that a traveler will
38 board a vehicle with longer expected travel time to the destination if that vehicle arrives first and the expected time
39 conditional upon the arrival of the vehicle is less than the expected time of arrival on a faster bus that has not arrived
40 yet.

41 For example, suppose that a traveler is waiting at a bus stop with bus vehicle runs between that stop and
42 their destination stop (local buses, express buses, etc.) When each relevant bus arrives, the passenger will make the
43 decision to board it based on the expected travel time for that run, along with the certain knowledge of no further
44 wait time, compared to their knowledge of the run times for subsequent runs and their best guess (which may be
45 informed by real-time signage) of the wait time for those buses.

46 The common lines issue is a problem in route choice modeling for a couple reasons. First, in a static model
47 context, this user behavior needs to be represented or flows and expected travel times will be inaccurate. Second,
48 this behavior can affect estimation. If a model does not represent this behavior and is used to generate paths for
49 model estimation, biased parameters could result.

1 *What work has been done already?*

2 Spiess and Florian (7) addressed the common lines problem by having passengers use a strategy to select from a
 3 route amongst a set of desirable routes at each transfer point in a static model. Nguyen and Pallottino translated this
 4 into graph-theoretic language, naming this construct as the hyperpath framework, which is the work upon which
 5 Fast-Trips is built. Cominetti and Correa (19) expanded on the lowest cost hyperpath strategy in the context of
 6 congestion, which affects wait times and the flow distribution. Hamdouch and Lawphongpanich (20) extended the
 7 hyperpath model to include capacity constraints in a dynamic context, where some passengers fail to board crowded
 8 vehicles and must wait to board. Hamdouch et al. (21) expanded on this more, differentiating between sitting and
 9 standing passengers and their corresponding comfort levels.

10 In terms of representing optimizing strategies, since Fast-Trips (and the original FAST-Trips) is a
 11 hyperpath-based model, it captures the behavioral aspect of a traveler considering catching the first vehicle if it
 12 might result in a lower cost overall trip. However, the issues that arise in biased estimated coefficients may still be
 13 problematic if the input schedule is not perfect with respect to the estimation dataset. Since this is a real issue in the
 14 Fast-Trips estimation due to the lack of precise data on vehicle arrival and departure times for the dates
 15 corresponding to survey data, one approach could be to look at the distribution of vehicle times in transit GPS data.
 16 Some form of this distribution could be applied to the network for estimation to overcome the biases inherent to
 17 misrepresenting transit schedules for those runs corresponding to the survey dataset.

18 **Stochastic Vehicle Arrivals**

19 *What's the problem? Why is this important?*

20 Related to the common lines problem, stochastic vehicle arrivals are an issue in and of themselves because they
 21 influence traveler behavior. Real world transit vehicle arrivals are stochastic for a variety of reasons, including
 22 traffic congestion, driver behavior, passenger-based delays (questions, fare discussions), dwell delays for boarding
 23 and alighting, vehicle problems, events or incidents, etc. A model that has no representation for these issues will
 24 therefore represent an inaccurate schedule and fail to reflect realistic user behavior. The common lines strategy is
 25 one component of this behavior, but so is the preference of a particular transit line, route or system for its reliability.

26 For example, suppose that a traveler has two options for getting from their origin to their destination. Both
 27 options start with route A to a given transfer point, but from there, the traveler has two options to their destination:
 28 route B which takes 20 minutes and route B Express which takes 5 minutes. Further, assume the following schedules
 29 for these buses:

- 30 ● Route A: origin to transfer, 9:30-10:00
- 31 ● Route B run 1: transfer to destination, 10:05-10:25
- 32 ● Route B run 2: transfer to destination, 10:15-10:35
- 33 ● Route B Express: transfer to destination, 10:15-10:20

34 Assume that the traveler has a simple utility function, $U = \beta_{IVT} \text{InVehicleTime} + \beta_{Wait} \text{WaitTime}$, where
 35 time is measured in minutes, $\beta_{IVT} = -0.1$ and $\beta_{Wait} = -0.2$. However, since the world does not function with perfect
 36 regularity, Route A may run late. Suppose that 25% of the time, it arrives at the transfer point too late to catch the
 37 first run of Route B. Then, the probabilities associated with each option at the transfer point are:

38
 39

TABLE I. Probabilities associated with transfer point alternative.

	Route A arrives on time	Route A arrives late	Combined Probability
Route B run 1	57%	0%	43%
Route B run 2	8%	18%	10%
Route B Express	35%	82%	47%

40

41 If a revealed choice dataset were used for estimation without any understanding of this built into the input
 42 schedule (meaning perfect accuracy, including vehicle delay) or the path generation, then the combined observed

1 choice would follow the distributions of the combined probability. Using these to estimate a route choice model
2 however would result in a biased estimate of β_{VT} and β_{wait} . In testing this particular example, the β_{wait} was
3 estimated at a biased value of -0.14 instead of -0.2, a significant difference.

4 *What work has been done already?*

5 While the common lines research is also relevant to stochastic vehicle arrivals, the Fast-Trips team does not know of
6 any publications that explicitly discuss representing stochastic vehicle arrivals in a dynamic transit assignment
7 model.

8 **WHAT'S NEXT?**

9 In light of the challenges mentioned above, the Fast-Trips Implementation team has decided that spending the time
10 and resources to estimate a model at this point would be premature. Instead, the remainder of the project will be split
11 into two work plans. On the research track, the team will continue to explore solutions and workarounds to these
12 challenges, within and possibly outside of, the Fast-Trips hyperpath framework. On the implementation track, the
13 Fast-Trips team will assert parameters for the Fast-Trips model based on prior research estimating mode choice
14 models in the San Francisco bay area. Next, the team will calibrate the model, adjusting configuration parameters,
15 base year networks and demand, exploring some form of distributing vehicle arrival times in this process. The team
16 will develop both metrics and performance targets around path-based quality of service, path set coverage, path
17 flows, and computational efficiency. These metrics and targets will also serve as valuable guides further down when
18 the research track bears fruit.

19 **CONCLUSION**

20 Over the last 18 months, as the multi-agency Fast-Trips Implementation team has worked to develop a dynamic
21 transit assignment model that generates a quality choice set and accurately represents the route choice probabilities,
22 a number of challenges have arisen which were not anticipated at the project's outset. Incorporating non link-
23 additive costs is difficult, and may not be possible to do in a mathematically consistent way. This is especially true
24 with complex fare structures in an extensive network in a reasonable run time. Accounting for route overlap is a
25 problem that has been well documented, but research is still ongoing in both the more straight-forward road-based
26 network context but especially in the dynamic transit space. Although strategies representing a traveler decision to
27 board a vehicle at a stop served by multiple common transit lines have been published and implemented, these
28 choices can bias estimation due to the lack of perfect knowledge of the choices faced by travelers in observed data
29 sets. Finally, representing vehicle arrival stochasticity from various sources, both modeled and exogenous, is
30 difficult but would also contribute greatly to better model estimation and representation of travel conditions and
31 outcomes. The Fast-Trips team will continue to follow the research on all of these problems mentioned, and
32 hopefully incorporate the best practices to the developing dynamic model.

33

34 **ACKNOWLEDGEMENTS**

35 The authors would like to thank the Federal Highway Program SHRP2 Implementation Assistance Program which
36 funded this work as well as the Fast-Trips Implementation Management Team and project manager: David Ory,
37 Billy Charlton, Joe Castiglione, Mark Simonson, and Diana Dorinson.

38

39 **REFERENCES**

- 40 1. Chiu, Y., J. Bottom, M. Mahut, A. Paz, R. Balakrishna, T. Waller, and J. Hicks. Dynamic Traffic
41 Assignment: A Primer. Transportation Research Circular E-C153, Washington, DC, 2011, pp 3-5.
42 2. Wilson, N. and A. Nuzzolo (editors). Schedule-Based Dynamic Transit Modeling: Theory and
43 Applications. Springer Science+Business Media, New York, 2004.
44 3. Khani, A., M. Hickman, and H. Noh. Trip-Based Path Algorithms Using the Transit Network Hierarchy.
45 In *Networks and Spatial Economics*, September 2015, Volume 15, Issue 3, pp 635-653.
46 4. Nguyen S, and S. Pallottino. Equilibrium Traffic Assignment for Large-Scale Transit Networks. In
47 *European Journal of Operational Research*, Vol. 37, No. 2, 1988, pp. 176-186.
48 5. Nguyen S, S. Pallottino, and M. Gendreau. Implicit Enumeration of Hyperpaths in a Logit Model for
49 Transit Networks. In *Transportation Science*, Vol. 32, No. 1, 1998, pp. 54-64.

- 1 6. Nguyen S, S. Pallottino, F. Malucellil. A Modeling Framework for Passenger Assignment on a Transport
2 Network with Timetables. In *Transportation Science*, Vol. 35, No. 3, 2001, pp. 238–249.
- 3 7. Spiess, H. and M. Florian. Optimal strategies: A new assignment model for transit networks. In
4 *Transportation Research Part B*, Vol. 23, 1989, pp. 83-102.
- 5 8. Constantin, I. and D. Florian. Journey Levels in Strategy-Based Transit Assignment: Modeling Integrated
6 Transit Fares and More. Presented at the 95th Annual Meeting of the Transportation Research Board,
7 Washington, D.C., 2016.
- 8 9. PTV Group, *PTV Visum Functions*, <http://vision-traffic.ptvgroup.com/en-us/products/ptv-visum/functions/>.
9 Accessed July 28, 2016.
- 10 10. Caliper, *TransCAD Transportation Planning Software: Planning and Travel Demand with TransCAD*.
11 <http://www.caliper.com/TCTravelDemand.htm>. Accessed July 28, 2016.
- 12 11. Zorn, L., E. Sall, and D. Wu. Incorporating crowding into the San Francisco Activity-Based Travel Model.
13 In *Transportation*, Vol. 39, No. 4, 2012, pp. 755-771.
- 14 12. Fosgerau, M., E. Frejinger, and A. Karlstrom. A Link Based Network Route Choice Model with
15 Unrestricted Choice Set. In *Transportation Research Part B: Methodological*, Vol 56, 2013, pp. 70-80.
- 16 13. Lo, H.K., C.W. Yip, and K.H. Wan. Modeling transfer and non-linear fare structure in multi-modal
17 network. In *Transportation Research Part B: Methodological*, Vol. 37, No. 2, 2003, pp. 149-170.
- 18 14. Cascetta, E, A. Nuzzolo, F. Russo, and A. Vitetta. A Modified Logit Route Choice Model Overcoming
19 Path Overlapping Problems. Specification and Some Calibration Results for Interurban Networks.
20 Presented at the 13th International Symposium on Transportation and Traffic Theory, Lyon, France, July
21 24-26, 1996.
- 22 15. Ben-Akiva, M. and M. Bierlaire. Discrete choice methods and their applications to short term travel
23 decisions. In *Handbook of Transportation Science* (R. W. Hall, ed.), Kluwer, Dordrecht, Netherlands, 1999,
24 pp. 5–33.
- 25 16. Frejinger, E., M. Bierlaire, and M. Ben-Akiva. Sampling of alternatives for route choice modeling. In
26 *Transportation Research Part B: Methodological*, Vol. 43, No. 10, 2009, pp. 984-994.
- 27 17. Hoogendoorn-Lanser, S., R. Nes, and P. Bovy. Path Size Modeling in Multinomial Route Choice Analysis.
28 In *Transportation Research Record: Journal of the transportation Research Board, No 1921*,
29 Transportation Research Board of the National Academies, Washington, D.C., 2005, pp. 27-34.
- 30 18. Chriqui, C. and P. Robillard. Common bus lines, *Transportation Science*, Vol. 9, No. 2, 1975, pp. 115-121.
- 31 19. Cominetti, R. and J. Correa. Common-Lines and Passenger Assignment in Congested Transit Networks. In
32 *Transportation Science*, Vol. 35, No.3, 2001, pp. 250-267.
- 33 20. Hamdouch, Y. and S. Lawphongpanich, S. Schedule-based transit assignment model with travel strategies
34 and capacity constraints. In *Transportation Research Part B*, Vol. 42, No. 7-8, 2008, pp. 663-684.
- 35 21. Hamdouch, Y., H. W. Ho, A. Sumalee, and G. Wang. Schedule-based Transit Assignment Model with
36 Vehicle Capacity and Seat Availability. In *Transportation Research Part B*, Vol 45, No. 10, 2011, pp.
37 1805-1830.